

# SPONTANEOUS COMBUSTION OF STORED SUGAR CANE BAGASSE

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## Abstract

Bagasse, the fibrous matter that remains after the extraction process from sugarcane, is still a valuable resource to the Sugar Milling Industry, since it may be used to generate electricity or as a building material. However, it is vulnerable to spontaneous combustion during storage. The Sugar Milling Research Institute in KwaZulu Natal is interested in finding methods and conditions that allow the survival of bagasse stockpiles without combustion. Provided a reasonable set of storage guidelines can be defined the SMRI can then investigate strategies for bagasse storage and subsequent use for electricity generation.

## 1 Introduction

Sugar milling is an important industry in South Africa which combines the agricultural aspect of growing sugar cane with the manufacture of refined sugar. The

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fibre residue resulting from the process of extracting sugar from the shredded cane is known as bagasse. In other countries the bagasse has been used as a fuel in the factory boilers for co-generation of steam and electricity. This obviously reduces costs and so improves competitiveness. Unfortunately, it is well-known that large piles of bagasse are prone to spontaneous combustion.

The Sugar Milling Research Institute (SMRI) situated in KwaZulu Natal is interested in storing bagasse for use in their furnaces, but due to obvious safety issues they would first like to understand the processes behind spontaneous combustion. This was the problem presented at the Mathematics in Industry Study Group meeting in 2016 (MISG2016) at the University of the Witwatersrand. Specifically three issues were raised by the institute which are critical in finding safe methods for bagasse storage and avoiding spontaneous combustion:

- (i) Calculating the maximum height of the bagasse heap to avoid spontaneous combustion,
- (ii) Investigating whether or not there are advantages in adjusting the moisture content,
- (iii) Investigating whether or not there is an advantage in pelletizing the bagasse.

Spontaneous combustion has been observed in a number of other industries and consequently there is a rich literature on the topic. For our study we focussed on a model developed by Gray *et al.* [1, 2, 3, 4]. In the following section this model will be explained and placed in the context of the problem presented to us by SMRI. Section 3 focusses on a coupled steady state problem which when analysed gives information that leads to the detailed process of non-dimensionalising of the governing equations in Section 4. The resulting governing equations are significantly simplified. More accurate models are discussed in Section 5. We then give concluding remarks in Section 6.

## 2 Mathematical model

The first recorded spontaneous combustion incident took place in the Mourilyan stockpile in 1983. This incident motivated experiments, some of which were reported in [1, 2] in 1984, that attempted to find out why bagasse would spontaneously combust and which conditions led to this phenomenon. Following two more bagasse ignition incidents between 1983 and 1988, Dixon [3] investigated further the process of spontaneous combustion of bagasse and found that moisture content in the bagasse plays a very significant role. Recommendations were therefore made from the latter study that the effect of moisture content should never be neglected in the mathematical modelling of the spontaneous combustion of bagasse stockpiles. Following these recommendations, Gray *et al.* [4] considered a mathematical model of the process of spontaneous combustion in bagasse which took the effect of moisture content into account. This paper laid the groundwork for the discussions and mathematical models analysed during MISG2016. In the study, Gray considered a

one dimensional model where the temperature  $U$ , the molar concentration of liquid water  $X$ , the water vapour  $Y$  and the oxygen content  $W$  all depend on the time  $t$  and the distance  $x$  measured from the bottom to the top of the stockpile. The model is given by the following four equations:

$$(\rho_b c_b + m_w X c_w) \frac{\partial U}{\partial t} = \kappa \nabla^2 U + Q \rho_b Z W \exp(-E/RU) + Q_w \rho_b Z_w X W \exp(-E_w/RU) f(U) + L_v [Z_c Y - Z_e X \exp(-L_v/RU)], \quad (1)$$

$$\frac{\partial Y}{\partial t} = D_Y \nabla^2 Y + Z_e X \exp(-L_v/RU) - Z_c Y, \quad (2)$$

$$\frac{\partial X}{\partial t} = -Z_e X \exp(-L_v/RU) + Z_c Y, \quad (3)$$

$$\frac{\partial W}{\partial t} = D_w \nabla^2 W - F \rho_b Z W \exp(-E/RU) - F \rho_b Z_w X W \exp(-E_w/RU) f(U), \quad (4)$$

where the function

$$f(U) = \left[ \frac{\tanh[0.6(58 - U + 273)] + 1}{2} \right], \quad (5)$$

was obtained from experiments [5] and the values for all the remaining unknown parameters in (1) to (4) with their respective units are given by Gray [4] and illustrated in Table 1. The nonlinear diffusion equation for temperature given by (1) involves a number of source terms. The first two sources show that heat generation from the dry and wet reactions follow the standard Arrhenius form. The final term shows that heat release or absorption, due to latent heat, is proportional to the rate of change of liquid. The mass balance equations (2) and (3) describe the variation of moisture, either as liquid or vapour, in the bagasse. Equation (2) shows that vapour can diffuse through the bagasse. The amount of vapour increases due to condensation of water and decreases due to evaporation. The liquid water, equation (3), is not free to diffuse since it will attach to the bagasse or accumulate at the bottom of the pile, so it simply interchanges mass with the vapour phase.

At temperatures less than  $58^\circ\text{C}$ , oxygen levels in the bagasse are low and they rapidly increase for temperatures greater than  $58^\circ\text{C}$  as shown in (4) and (5).

The function  $f(U)$  in (5) acts as a switch. Below  $U = 58 + 273\text{K}$ ,  $f(U)$  is approximately 1; there is a rapid transition to 0 as  $U$  approaches  $58 + 273\text{K}$  and so we may assume that (5) takes the form

$$f(U) = \begin{cases} 1, & U < 58 + 273\text{K}, \\ 0, & U \geq 58 + 273\text{K}. \end{cases} \quad (6)$$

For temperatures  $U < 58^\circ\text{C}$  the overall reaction is driven by the moisture dependent reaction, whereas for temperatures  $U \geq 58^\circ\text{C}$  the moisture dependent effect vanishes and the overall reaction is then driven by oxidation.

<b>Nomenclature</b>	
<i>Constants</i>	
$E$	activation energy of dry reaction, $1.08 \times 10^5 \text{ J mol}^{-1}$
$E_w$	activation energy of wet reaction, $6.5 \times 10^4 \text{ J mol}^{-1}$
$R$	universal gas constant, $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
$Q$	exothermicity of dry reaction, $1.7 \times 10^7 \text{ J kg}^{-1}$
$Z$	pre-exponential factor of the dry reaction, $2.7 \times 10^7 \text{ m}^3 \text{ mol}^{-1} \text{ s}^{-1}$
$Q_w \rho_b Z_w$	coefficient of the wet reaction, $1.683 \times 10^8 \text{ J m}^3 \text{ s}^{-1} \text{ mol}^{-2}$
$Z_e$	pre-exponential factor of evaporation, $3.41 \times 10^4 \text{ s}^{-1}$
$Z_c$	pre-exponential factor of condensation, $4.7 \text{ s}^{-1}$
$L_v$	latent heat of vaporization, $42 \times 10^3 \text{ J mol}^{-1}$
$\kappa$	thermal conductivity, $0.5 \text{ J m}^{-1} \text{ K}^{-1} \text{ s}^{-1}$
$D_Y, D_W$	diffusivities of water vapour and oxygen resp., $2.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$
$h$	heat transfer coefficient, $5 \text{ J m}^{-2} \text{ K}^{-1} \text{ s}^{-1}$
$h_Y, h_W$	water vapour and oxygen transfer coefficients resp., $5 \text{ m s}^{-1}$
$m_w$	mass of water, $18 \times 10^{-3} \text{ kg mol}^{-1}$
$\rho_w$	density of water, $1 \times 10^3 \text{ kg m}^{-3}$
$\rho_b$	density of dry bagasse, $125 \text{ kg m}^{-3}$
$c_b$	heat capacity of dry bagasse, $1.4 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$
$c_w$	heat capacity of water, $4.19 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$
$Y_a$	ambient water vapour concentration, $1.74 \text{ mol m}^{-3}$
$W_a$	oxygen concentration in air, $8.04 \text{ mol m}^{-3}$
$F$	moles of $\text{O}_2$ consumed per kg of bagasse, $33.33 \text{ mol kg}^{-1}$
<i>Variables</i>	
$p$	% increase in water content
$T$	temperature, K
$t$	time, s
$X$	liquid water, $\text{mol m}^{-3}$
$Y$	water vapour, $\text{mol m}^{-3}$
$W$	oxygen, $\text{mol m}^{-3}$

Table 1: *Nomenclature and values of various constants with their respective units, taken from [4]. The temperature  $T$  in this table is replaced by the symbol  $U$  in the present work.*

While Gray considered the Newton cooling boundary condition on both ends of the bagasse, such that a symmetrical domain is achieved, we only consider the same boundary condition at the top surface,  $x = L$ ,

$$-\kappa \frac{\partial U}{\partial x} = h(U - U_a), \quad -D_Y \frac{\partial Y}{\partial x} = h_Y(Y - Y_a), \quad -D_W \frac{\partial W}{\partial x} = h_W(W - W_a). \quad (7)$$

The bagasse is placed on a flat surface with negligible conductive properties. We therefore assume that the bottom is completely insulated such that the no flow condition (of heat or material) at  $x = 0$  is imposed:

$$\frac{\partial U}{\partial x} = 0, \quad \frac{\partial Y}{\partial x} = 0, \quad \frac{\partial W}{\partial x} = 0. \quad (8)$$

The initial conditions are

$$U(x, 0) = U_0(x), \quad Y(x, 0) = Y_0(x), \quad X(x, 0) = X_0(x), \quad W(x, 0) = W_0(x). \quad (9)$$

In this work, we focus on situations close to ignition, so “investigating the worst case scenario”.

### 3 Steady state problem

We begin the analysis by considering the steady-state equations. This is useful not only for understanding the large time behaviour, but also to determine the appropriate scaling for the non-dimensionalisation in order to examine the bifurcation diagram.

First consider the steady equations for  $X$  and  $Y$ :

$$0 = D_Y \frac{\partial^2 Y}{\partial x^2} + Z_e X \exp\left(-\frac{L_v}{RU}\right) - Z_c Y, \quad (10)$$

$$0 = -Z_e X \exp\left(-\frac{L_v}{RU}\right) + Z_c Y. \quad (11)$$

Adding the two equations determines  $Y_{xx} = 0$  and, after applying the boundary conditions we find  $Y_s = Y_a$  (where the subscript  $s$  denotes steady-state). Using equation (11) we may then write down an expression for the liquid concentration as

$$X_s = \frac{Z_c Y_a}{Z_e} \exp\left(\frac{L_v}{RU}\right). \quad (12)$$

Note that the steady-state for  $X$  varies with position,  $x$ , due to the temperature in the exponential.

The terms in equation (11) represent conservation of liquid and vapour. The latent heat term in equation (1) represents the energy resulting from the liquid vaporising and the vapour condensing. In the steady-state, according to equation (11), these terms balance and so the heat equation (1) reduces to

$$0 = \kappa \frac{\partial^2 U}{\partial x^2} + Q_{\rho_b} Z W \exp\left(-\frac{E}{RU}\right) + Q_{w\rho_b} Z_w X_s W \exp\left(-\frac{E_w}{RU}\right) f(U). \quad (13)$$

The oxygen equation is

$$0 = D_W \frac{\partial^2 W}{\partial x^2} - F \rho_b Z W \exp\left(-\frac{E}{RU}\right) - F \rho_b Z_w X W \exp\left(-\frac{E_w}{RU}\right) f(U). \quad (14)$$

Our interest lies in the situation where spontaneous combustion is likely so we will focus on the high temperature regime (everywhere above 58°C), consequently we may neglect the terms involving  $f(U)$ . The above equations may then be combined to give

$$\frac{\kappa}{Q} \frac{\partial^2 U}{\partial x^2} + \frac{D_W}{F} \frac{\partial^2 W}{\partial x^2} = 0. \quad (15)$$

After integrating and applying the boundary conditions at  $x = 0$  we find

$$\frac{\kappa}{Q} U + \frac{D_W}{F} W = C_0. \quad (16)$$

This shows that there is linear relation between temperature and oxygen content, that is, as the temperature increases, the oxygen concentration decreases and vice-versa.

In the following section we will non-dimensionalise the model using the steady-state solutions as a guide. First, we have obtained  $Y_s = Y_a$ , which will be our vapour scale. The liquid steady-state is dependent on  $x$ . Since we are interested in the ignition of the bagasse a sensible  $X$  scale is

$$\Delta X = \frac{Z_c Y_a}{Z_e} \exp\left(\frac{L_v}{RU_i}\right), \quad (17)$$

where  $U_i$  is the ignition temperature.

## 4 Mathematical model in dimensionless form

In this section, we non-dimensionalise the governing equations (1) to (4) with their corresponding boundary conditions (7) and (8) and the initial conditions (9). We first introduce the dimensionless variables

$$\bar{U} = \frac{U - U_a}{\Delta U}, \quad \bar{x} = \frac{x}{L}, \quad \bar{t} = \frac{t}{\Delta t}, \quad \bar{Y} = \frac{Y}{\Delta Y}, \quad \bar{W} = \frac{W}{\Delta W}, \quad \bar{X} = \frac{X}{\Delta X}. \quad (18)$$

The characteristic temperature is chosen to be

$$\Delta U = U_i - U_a, \quad (19)$$

where  $U_i$  is the ignition temperature and  $U_a$  is the ambient temperature, and the characteristic height of the stockpile is  $L$ . Balancing the first and second terms of equation (1) gives the diffusion time scale

$$\Delta t = \frac{L^2(\rho_b c_b + m_w c_w \Delta X)}{\kappa} = \frac{L^2}{D_U}, \quad D_U = \frac{\kappa}{(\rho_b c_b + m_w c_w \Delta X)}. \quad (20)$$

We choose the characteristic vapour scale to be the vapour concentration at ambient conditions  $\Delta Y = Y_a$  and the characteristic oxygen content to be the oxygen at ambient conditions  $\Delta W = W_a$ . The characteristic liquid content  $\Delta X$  is given by equation (17).

Expressing (3) in dimensionless parameters gives

$$\frac{\Delta X}{\Delta t} \frac{\partial \bar{X}}{\partial \bar{t}} = -Z_e \Delta X \bar{X} \exp \left[ -\frac{L_v}{R(U_a + \Delta U \bar{U})} \right] + Z_c \Delta Y \bar{Y}. \quad (21)$$

This may be rewritten as

$$\frac{1}{\Delta t Z_e} \exp \left( \frac{L_v}{R U_i} \right) \frac{\partial \bar{X}}{\partial \bar{t}} = -\bar{X} \exp \left( \frac{L_v}{R U_i} - \frac{L_v}{R[U_a + \Delta U \bar{U}]} \right) + \bar{Y}. \quad (22)$$

Using the values of Table 1 we find that the coefficient of the left side of (22) is of the order  $\mathcal{O}(10^{-5})$  and so we may neglect the time derivative. This fits with the observation that the moisture reaction is quite rapid (on the order of days) while the entire storage time for bagasse may be around 9 months. This is verified by the time evolution of temperature obtained from experiments in [5] which show that there is a sharp increase in temperature for the first ten days after which the temperature stabilizes over approximately the next 200 days. Following a gradual temperature drop, stability is reached again after 350 days. This observation allows us to express  $\bar{X}$  in terms of  $\bar{U}$  and  $\bar{Y}$

$$\bar{X} = \exp \left( -\frac{\alpha_{L_v} (\bar{U} - 1)}{1 + \frac{\Delta U}{U_i} (\bar{U} - 1)} \right) \bar{Y}, \quad (23)$$

where

$$\alpha_{L_v} = \frac{L_v \Delta U}{R U_i^2} = \mathcal{O}(1). \quad (24)$$

As with the steady-state analysis it follows that the last two terms in the heat and vapour equations (1) and (2) vanish so that they reduce to

$$\begin{aligned} (\rho_b c_b + m_w X c_w) \frac{\partial U}{\partial t} &= \kappa \frac{\partial^2 U}{\partial x^2} + Q \rho_b Z W \exp(-E/RU) \\ &+ Q_w \rho_b Z_w X W \exp(-E_w/RU) f(U), \end{aligned} \quad (25)$$

$$\frac{\partial Y}{\partial t} = D_Y \frac{\partial^2 Y}{\partial x^2}. \quad (26)$$

In dimensionless form these two equations are

$$\begin{aligned} (\beta_1 + \beta_2 \bar{X}) \frac{\partial \bar{U}}{\partial \bar{t}} &= \frac{\partial^2 \bar{U}}{\partial \bar{x}^2} + A_E \bar{W} \exp \left[ \frac{\alpha_E (\bar{U} - 1)}{1 + \frac{\Delta U}{U_i} (\bar{U} - 1)} \right] \\ &+ A_{E_w} \bar{X} \bar{W} \exp \left[ \frac{\alpha_{E_w} (\bar{U} - 1)}{1 + \frac{\Delta U}{U_i} (\bar{U} - 1)} \right] f(\bar{U}), \end{aligned} \quad (27)$$

$$\kappa_Y \frac{\partial \bar{Y}}{\partial \bar{t}} = \frac{\partial^2 \bar{Y}}{\partial \bar{x}^2}, \quad (28)$$

where

$$\beta_1 = \frac{\rho_b c_b L^2}{\kappa \Delta t} = \mathcal{O}(1), \quad \beta_2 = \frac{m_w c_w \Delta X L^2}{\kappa \Delta t} = \mathcal{O}(10^{-1}), \quad (29)$$

$$A_E = \frac{Q \rho_b Z \Delta W L^2}{\kappa \Delta U} \exp\left(-\frac{E}{R U_i}\right) = \mathcal{O}(1), \quad (30)$$

$$A_{E_w} = \frac{Q_w \rho_b Z_w \Delta X \Delta W L^2}{\kappa \Delta U} \exp\left(-\frac{E_w}{R U_i}\right) = \mathcal{O}(10^2), \quad (31)$$

$$\alpha_E = \frac{E \Delta U}{R U_i^2} = \mathcal{O}(1), \quad \alpha_{E_w} = \frac{E_w \Delta U}{R U_i^2} = \mathcal{O}(1), \quad \kappa_Y = \frac{L^2}{\Delta t D_Y} = \mathcal{O}(10^{-1}). \quad (32)$$

The dimensionless form of the oxygen equation (4) is

$$\begin{aligned} \kappa_W \frac{\partial \bar{W}}{\partial \bar{t}} = \frac{\partial^2 \bar{W}}{\partial \bar{x}^2} - B_E \bar{W} \exp\left[\frac{\alpha_E (\bar{U} - 1)}{1 + \frac{\Delta U}{U_i} (\bar{U} - 1)}\right] \\ - B_{E_w} \bar{X} \bar{W} \exp\left[\frac{\alpha_{E_w} (\bar{U} - 1)}{1 + \frac{\Delta U}{U_i} (\bar{U} - 1)}\right] f(\bar{U}), \end{aligned} \quad (33)$$

where

$$\kappa_W = \frac{L^2}{\Delta t D_W} = \mathcal{O}(10^{-1}), \quad B_E = \frac{F \rho_b Z L^2}{D_W} \exp\left(-\frac{E}{R U_i}\right) = \mathcal{O}(1), \quad (34)$$

$$B_{E_w} = \frac{F \rho_b Z_w \Delta X L^2}{D_w} \exp\left(-\frac{E_w}{R U_i}\right) = \mathcal{O}(10). \quad (35)$$

The boundary conditions (7) and (8) in dimensionless variables are

$$\bar{x} = 1: \quad -\frac{\partial \bar{U}}{\partial \bar{x}} = \gamma \bar{U}, \quad -\frac{\partial \bar{Y}}{\partial \bar{x}} = \gamma_Y (\bar{Y} - 1), \quad -\frac{\partial \bar{W}}{\partial \bar{x}} = \gamma_W (\bar{W} - 1), \quad (36)$$

$$\bar{x} = 0: \quad \frac{\partial \bar{U}}{\partial \bar{x}} = 0, \quad \frac{\partial \bar{Y}}{\partial \bar{x}} = 0, \quad \frac{\partial \bar{W}}{\partial \bar{x}} = 0, \quad (37)$$

where

$$\gamma = \frac{hL}{k}, \quad \gamma_Y = \frac{h_Y L}{D_Y}, \quad \gamma_W = \frac{h_W L}{D_W}. \quad (38)$$

Note that  $\gamma = \mathcal{O}(10)$  and  $\gamma_Y = \gamma_W = \mathcal{O}(10^5)$  so we may simplify the last two boundary conditions in (36) to  $\bar{Y} = \bar{W} = 1$  at  $\bar{x} = 1$ . The initial conditions are

$$\bar{U} = \bar{U}_0, \quad \bar{Y} = \bar{Y}_0, \quad \bar{W} = \bar{W}_0, \quad \text{at } \bar{t} = 0, \quad (39)$$

where

$$\bar{U}_0 = \frac{U_0 - U_a}{\Delta U}, \quad \bar{Y}_0 = \frac{Y_0}{\Delta Y}, \quad \bar{W}_0 = \frac{W_0}{\Delta W}. \quad (40)$$



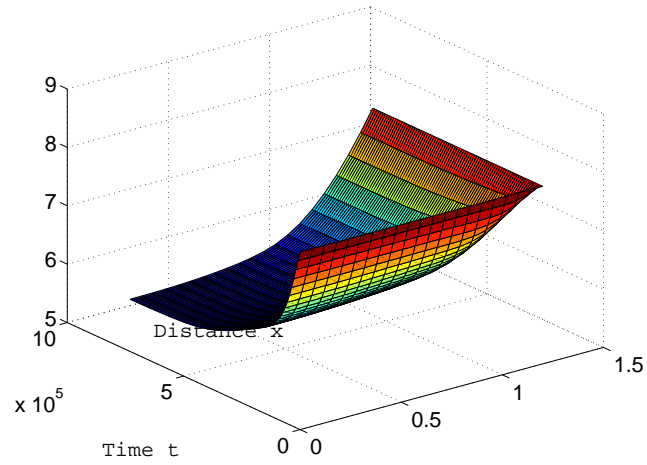


Figure 1: Surface plot of the oxygen,  $W$ .

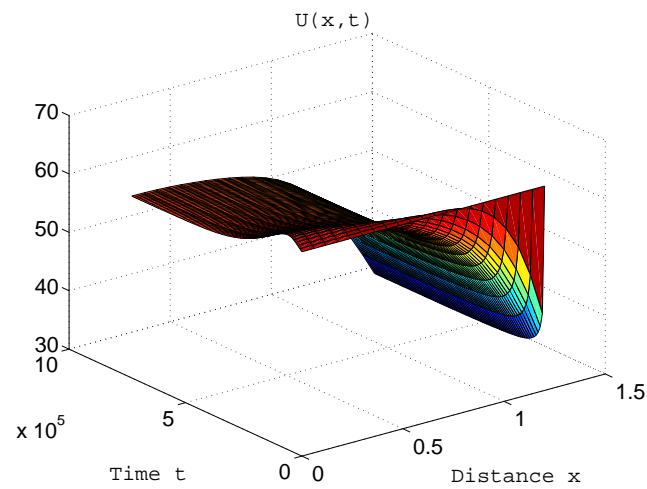


Figure 2: Surface plot of the temperature,  $U$ , with  $L = 1.2$ .

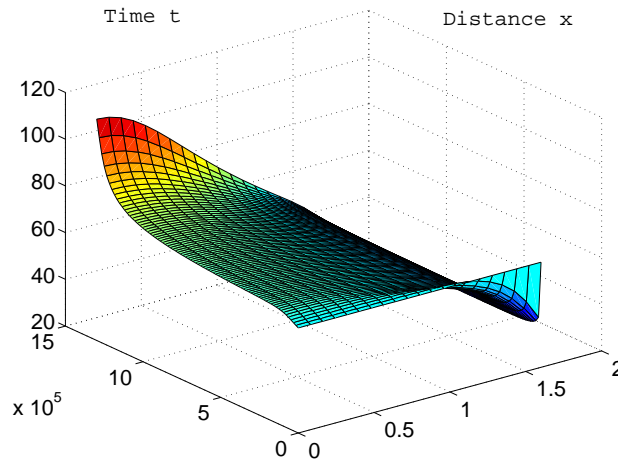


Figure 3: Surface plot of the temperature,  $U$ , with  $L = 1.6$ .

In Figures 1-3 we present results from this model. The axes are dimensional. The variation of the oxygen content in a 1.2m pile is shown in Figure 1. Initially the concentration is the same as the value of the air, namely  $8.04 \text{ mol/m}^3$ . It remains at this value at the top of the pile, however as time increases the value decreases elsewhere, with a minimum at the bottom of the pile, where it is hardest for the oxygen to diffuse. The corresponding temperature plot is shown in Figure 2. The initial condition on temperature is constant and set at a high value,  $60^\circ\text{C}$ , to focus on the dry reaction leading to ignition. After a brief period, where the temperature rises slightly near the bottom of the pile the temperature reduces everywhere and it is clear that ignition will never occur. The behaviour is consistent with the experiments of [5] who report a sharp rise in temperature for the first 10 days, followed by a stable period lasting approximately 200 days. In Figure 3 we show the temperature for a slightly larger pile, 1.6m high. At early times this shows a similar behaviour to the previous case, with an initial small rise in temperature near the bottom of the pile, this is followed by an almost steady-state, where the temperature is relatively independent of time. On the order of  $15 \times 10^6\text{s}$ , or 10 days, the temperature starts to rise in a manner where ignition is inevitable: dry bagasse ignites at approximately  $94^\circ\text{C}$ . The maximum temperature in this simulation is around  $110^\circ\text{C}$ . We may therefore conclude that given the ambient conditions used in the simulations bagasse piles must be kept below 1.6m.

## 5 A reduced model

As noted in a previous section, the liquid water content is well approximated by (23). This will lead to errors of order  $10^{-3}\%$ , hence it seems reasonable to replace  $\bar{X}$  using this expression. In general we should therefore describe the problem using three coupled equations, namely equations (27), (28), (33).

A further reduction was also examined during the meeting. This was based on the observation that the coefficients  $\kappa_W, \kappa_Y$  are order  $10^{-1}$ , so neglecting the corresponding time derivatives may lead to errors up to 10%. A rough approximation to the solution is then given by equation (23) linked to the pseudo-steady solution  $\bar{Y} = 1$  and

$$\begin{aligned} (\beta_1 + \beta_2 \bar{X}) \frac{\partial \bar{U}}{\partial t} &= \frac{\partial^2 \bar{U}}{\partial \bar{x}^2} + A_E \bar{W} \exp\left(\frac{\alpha_E (\bar{U} - 1)}{1 + \frac{\Delta U}{U_i} (\bar{U} - 1)}\right) \\ &\quad + A_{E_w} \bar{X} \bar{W} \exp\left(\frac{\alpha_{E_w} (\bar{U} - 1)}{1 + \frac{\Delta U}{U_i} (\bar{U} - 1)}\right) f(\bar{U}), \end{aligned} \quad (41)$$

$$\begin{aligned} 0 &= \frac{\partial^2 \bar{W}}{\partial \bar{x}^2} - B_E \bar{W} \exp\left(\frac{\alpha_E (\bar{U} - 1)}{1 + \frac{\Delta U}{U_i} (\bar{U} - 1)}\right) \\ &\quad - B_{E_w} \bar{X} \bar{W} \exp\left(\frac{\alpha_{E_w} (\bar{U} - 1)}{1 + \frac{\Delta U}{U_i} (\bar{U} - 1)}\right) f(\bar{U}). \end{aligned} \quad (42)$$

This reduced model may be simpler to solve than that of the previous section, however we did not have time to investigate it further.

## 6 Conclusions

A one dimensional model describing the dynamics in the bagasse stockpile has been considered, our primary source was the papers of Gray *et al* who published a large body of work on spontaneous combustion in bagasse and other materials. The model we employed incorporated the effects of moisture, liquid water and oxygen content on the temperature since the literature indicated these were the dominant effects. For temperatures  $U < 58 + 273K$ , the reaction is dominated by the moisture content and is quite rapid (on the order of days) while for temperatures  $U \geq 58 + 273K$ , the reaction is dominated by the oxygen content in the bagasse and the reaction is very slow (on the order of months).

The non-dimensionalisation showed that the liquid water content is always close to its steady-state value (with errors of the order  $10^{-3}\%$ ). This observation allows the removal of various terms in the governing equations and so simplifies the mathematical model. The simplified system, consisting of three partial differential equations was solved numerically. The results were consistent with behaviour reported from experiments, with an initial rise in temperature, followed by a relatively stable period. We carried out two simulations, the first on a 1.2m pile which indicated that ignition would never occur, the second, on a 1.6m pile, led to ignition.

At the start of the study group we were asked specific questions. Given the time constraints of the meeting it was impossible to reach the desired goals, which could clearly be the subject of a continued, more detailed study. The main achievement of the meeting was to develop a mathematical model that leads to realistic results. This will be invaluable for any continuation of the project. A specific question posed was to provide a set of simple guidelines for safe bagasse storage. A mathematical

model has now been identified which can be used to determine these guidelines. A second question was whether there are advantages in adjusting the moisture content of the pile. The model showed that there is a given height above which the pile will ignite and this height varies with ambient conditions and moisture content. The moisture content is particularly important, since water is required for a number of exothermic reactions in the model. Our preliminary results showed that an increase in moisture content in the bagasse can cause a previously stable stockpile to spontaneously combust. This result was also observed in the literature. This suggests that the moisture content of the bagasse should be kept to a minimum, but of course there are costs associated with the drying process. We also found that for any ambient condition, a bagasse stockpile can combust if the height is sufficiently large. Therefore the risk of combustion is directly proportional to the height of the stockpiles.

Further suggested work includes:

- Finalise the one-dimensional model, by examining it in more detail, with more simulations to determine suitable stockpile heights for given ambient conditions and also the effect of sudden changes to these conditions for example through rainfall.
- The model of Gray *et al* should be examined in detail, to verify all terms are correct. Should height variation of parameters be included, i.e. density increasing with depth. The boundary conditions should also be examined, for example is it realistic that no heat is lost at the substrate? Can the storage environment be adapted to lessen the risk of fire?
- The model could easily be extended to two dimensions, which would then allow lateral movement of heat, possibly leading to further cooling.

In summary we should point out that the model analysed during the week appears to be an excellent starting point in the study of bagasse storage but for such an important issue it is quite clear that a more detailed (and time-consuming) investigation must be undertaken.

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